CE 416
Prestressed Concrete Sessional
(Lab Manual)

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Preface

The idea of prestressed concrete has been developed around the latter decades of the 19th century, but its use was limited by the quality of the materials at that time. It took until the 1920s and ’30s for its materials development to progress to a level where prestressed concrete could be used with confidence. Currently many bridges and skyscrapers are designed as prestressed structures. This manual intends to provide a general overview about the design procedure of a two way post tensioned slab and a girder. To provide a complete idea, the stress computation, the reinforcement detailing, shear design, the jacking procedure etc. are discussed in details.

This Lab manual was prepared with the help of the renowned text book "Design of Prestressed Concrete Structures", 3rd Edition by T.Y. Lin and Ned H. Burns. The design steps for a two way post-tensioned slab was prepared according to the simple hand calculation provided by PCA (Portland Cement Association) as well as the ACI 318-05 code requirements. The design steps for a post-tensioned composite bridge girder were prepared with the help of several sample design calculation demonstrated in different PC structure design books and seminar papers. It has been done in accordance with AASHTO LRFD Bridge Design Specifications.

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1. INTRODUCTION

Prestressed concrete is a method for overcoming concrete's natural weakness in tension. It can be used to produce beams, floors or bridges with a longer span than is practical with ordinary reinforced concrete. Prestressing tendons (generally of high tensile steel cable or rods) are used to provide a clamping load which produces a compressive stress that balances the tensile stress that the concrete compression member would otherwise experience due to a bending load. Traditional reinforced concrete is based on the use of steel reinforcement bars, rebars, inside poured concrete.

Prestressing can be accomplished in three ways:

- Pre-tensioned concrete,
- Bonded or
- Unbonded post-tensioned concrete.

Pre-tensioned concrete

Pre-tensioned concrete is cast around already tensioned tendons. This method produces a good bond between the tendon and concrete, which both protects the tendon from corrosion and allows for direct transfer of tension. The cured concrete adheres and bonds to the bars and when the tension is released it is transferred to the concrete as compression by static friction. However, it requires stout anchoring points between which the tendon is to be stretched and the tendons are usually in a straight line. Thus, most pre-tensioned concrete elements are prefabricated in a factory and must be transported to the construction site, which limits their size. Pre-tensioned elements may be balcony elements, lintels, floor slabs, beams or foundation piles.

Bonded post-tensioned concrete

Bonded post-tensioned concrete is the descriptive term for a method of applying compression after pouring concrete and the curing process (*in situ*). The concrete is cast around a plastic, steel or aluminium curved duct, to follow the area where otherwise tension would occur in the concrete element. A set of tendons are fished through the duct and the concrete is poured. Once the concrete has hardened, the tendons are tensioned by hydraulic jacks that react against the concrete member itself. When the tendons have stretched sufficiently, according to the design specifications (see Hooke's law), they are wedged in position and maintain tension after the jacks are removed, transferring pressure to the concrete. The duct is then grouted to protect the tendons from corrosion. This method is commonly used to create monolithic slabs for house construction in locations where expansive soils (such as adobe clay) create problems for the typical perimeter foundation. All stresses from seasonal expansion and contraction of the underlying soil are taken into the entire tensioned slab, which supports the building without significant flexure. Post-tensioning is also used in the construction of various bridges, both after concrete is cured after support by falsework and by the assembly of prefabricated sections, as in the segmental bridge. The advantages of this system over un-bonded post-tensioning are:

1. Large reduction in traditional reinforcement requirements as tendons cannot distress in accidents.
2. Tendons can be easily 'weaved' allowing a more efficient design approach.
3. Higher ultimate strength due to bond generated between the strand and concrete.
4. No long term issues with maintaining the integrity of the anchor/dead end.

Un-bonded post-tensioned concrete

Un-bonded post-tensioned concrete differs from bonded post-tensioning by providing each individual cable permanent freedom of movement relative to the concrete. To achieve this, each individual tendon is coated with a grease (generally lithium based) and covered by a plastic sheathing formed in an extrusion process. The transfer of tension to the concrete is achieved by the steel cable acting against steel anchors embedded in the perimeter of the slab. The main disadvantage over bonded post-tensioning is the fact that a cable can distress itself and burst out of the slab if damaged (such as during repair on the slab). The advantages of this system over bonded post-tensioning are:

1. The ability to individually adjust cables based on poor field conditions (For example: shifting a group of 4 cables around an opening by placing 2 to either side).
2. The procedure of post-stress grouting is eliminated.
3. The ability to de-stress the tendons before attempting repair work.

Applications:

- Prestressed concrete is the predominating material for floors in high-rise buildings and the entire containment vessels of nuclear reactors.
- Un-bonded post-tensioning tendons are commonly used in parking garages as barrier cable. Also, due to its ability to be stressed and then de-stressed, it can be used to temporarily repair a damaged building by holding up a damaged wall or floor until permanent repairs can be made.
- The advantages of prestressed concrete include crack control and lower construction costs; thinner slabs - especially important in high rise buildings in

which floor thickness savings can translate into additional floors for the same (or lower) cost and fewer joints, since the distance that can be spanned by post-tensioned slabs exceeds that of reinforced constructions with the same thickness. Increasing span lengths increases the usable unencumbered floorspace in buildings; diminishing the number of joints leads to lower maintenance costs over the design life of a building, since joints are the major focus of weakness in concrete buildings.

- The first prestressed concrete bridge in North America was the Walnut Lane Memorial Bridge in Philadelphia, Pennsylvania. It was completed and opened to traffic in 1951. Prestressing can also be accomplished on circular concrete pipes used for water transmission. High tensile strength steel wire is helically-wrapped around the outside of the pipe under controlled tension and spacing which induces a circumferential compressive stress in the core concrete. This enables the pipe to handle high internal pressures and the effects of external earth and traffic loads.

Design Example of a Post-tensioned Composite Bridge Girder

General

This chapter demonstrates the detailed design and analysis of a 73 m span Pre-stressed Post-tensioned I/Bulb Tee Girder. An interior girder of a double lane bridge having total width of 9.8 m and carriage width of 7.3 m is considered as per our national standard of double lane highway. The design follows AASHTO LFRD Bridge Design Specifications and California Department of Transportation (CalTrans) Bridge Design Practice. (All dimensions are in mm unless otherwise stated)

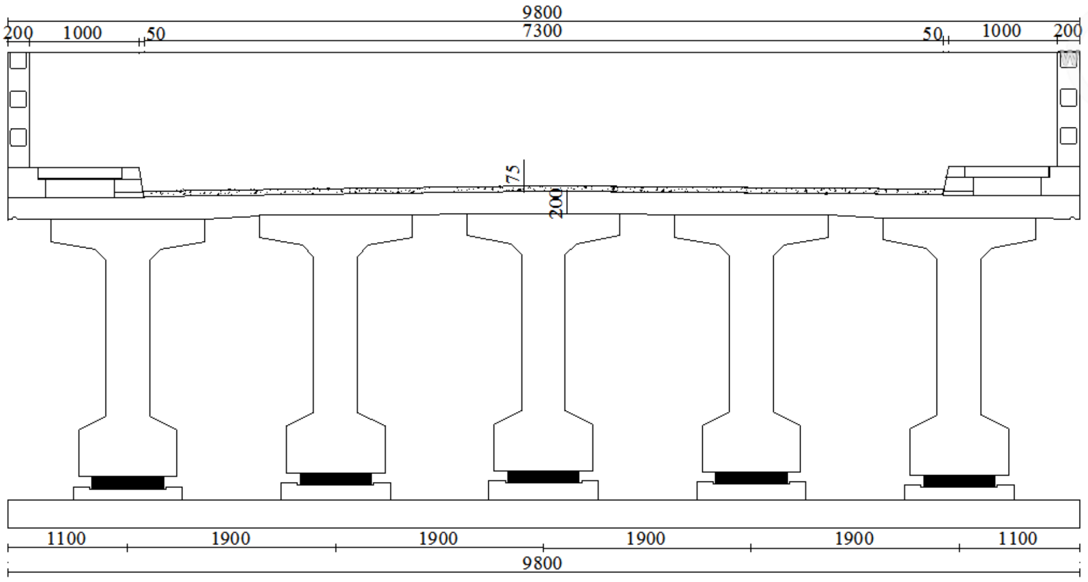


Figure 1 Cross-Section of Deck Slab and Girder

Specifications

Girder Details

Girder Location	=	Interior Girder
Girder Type	=	Post-tensioned I Girder (Cast-in-situ)
Overall Span Length	=	73 m
CL of Bearing	=	0.45 m
Effective Span Length	=	72.1 m
Girder Depth without Slab	=	3.5 m ≥ 0.045 x 73 – 0.2 (Deck) = 3.085 m [AASTHO `07, Table 2.5.2.6.3-1]
Nos. of Girder	=	5
Spacing of Main Girder	=	1.9m

Deck Slab

Thickness	=	0.2m
Total width	=	9.8m
Carriage way	=	7.3 m
Nos. of Lane	=	2
Thickness of WC	=	0.075 m

Cross Girder

Number of Cross Girder	=	10
Depth	=	3.2 m
Thickness of Interior Cross Girder	=	0.35 m
Thickness of Exterior Cross Girder	=	0.60 m

Concrete Material Properties

Strength of Girder Concrete, f'_c	=	45 MPa [28 days Cylinder Strength]
Strength at 1^{st} Stage, f'_{ci}	=	45x66% = 30 MPa [10-14 days]
Strength at 2^{nd} Stage, f'_{ci}	=	45x90% = 40 MPa [21 days]
Strength of Deck Slab	=	40 MPa [28 days Cylinder Strength]
Unit Weight of Concrete	=	24 KN/m^3
Unit Weight of WC	=	23 KN/m^3
MOE of Girder, E_c	=	$4800\sqrt{45}$ = 32200 MPa
MOE of Girder 1^{st} Stage, E_{ci}	=	$4800\sqrt{30}$ = 26290 MPa
MOE of Girder 2^{nd} Stage, E_{ci}	=	$4800\sqrt{40}$ = 30358 MPa
MOE of Deck Slab, E_c	=	$4800\sqrt{40}$ = 30358 MPa

Prestressing Material Properties

Anchorage Type	=	19K15
Strand Details	=	[15.24 mm dia. 7 Ply low relaxation]
Nos. of Strand	=	19
Ultimate Strength of Strand, f_{pu}	=	1860 MPa
Yield Strength, f_y	=	$0.9 f_{pu}$ = 1674 MPa
MOE of Strand, E_s	=	197000 MPa
Area of each Strand	=	140 mm^2
Area of each Cable	=	140 x 19 = 2660 mm^2
Jacking Force per Cable	=	1395 x 2660 = 3710 KN $\leq 0.9 f_y$ [AASTHO `07, Table 5.9.3-1]
Number of Cable Initially Assumed	=	9 Nos.
Cable in 1^{st} Stage	=	7 Nos.
Cable in 2^{nd} Stage	=	2 Nos.
Cable Orientation	=	Cable 3-9, 1^{st}& Cable 1-2, 2^{nd} Stage

Calculation of Section Properties
Non-Composite Section

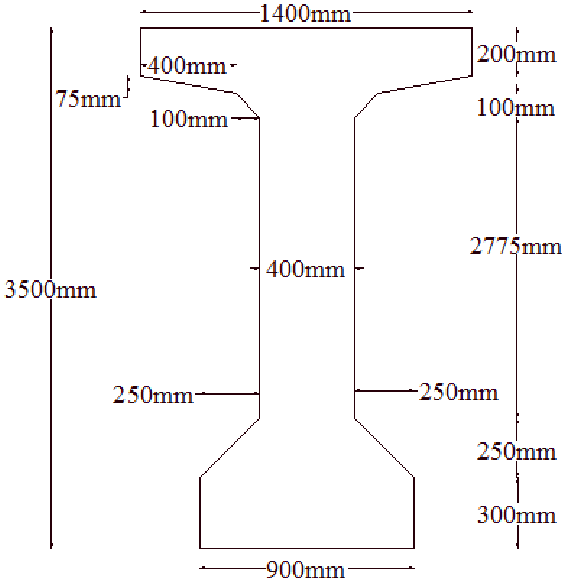


Figure -2 Non-Composite Section at Middle

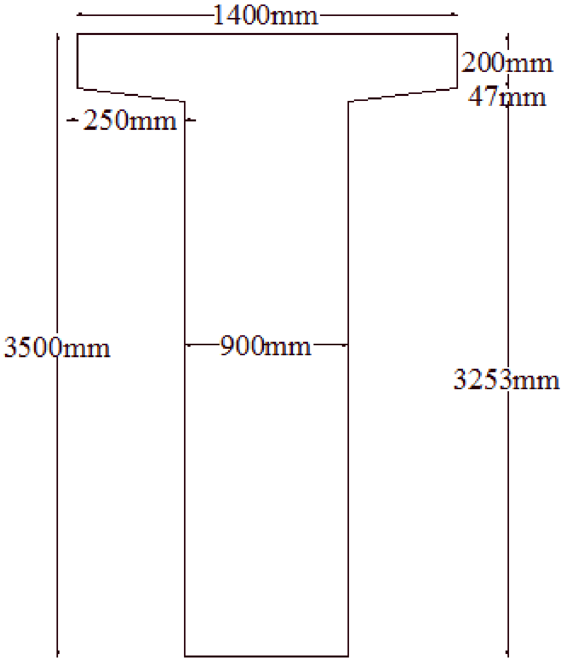


Figure -3 Non-Composite Section at End

Table -1 Section Property of Non-Composite

Part	Size	B	h	A	Y	Ay	Y_N	I	Y_b	AY_b^2
Unit		Mm	mm	mm^2	mm	mm^3	mm	mm^4	mm	mm^4
1		1400	200	$2.8x10^5$	3400	$0.952x10^9$		$9.33x10^8$	-1603	$7.2x10^{11}$
2		600	75	45000	3262	$1.47x10^8$		$2.11x10^7$	-1465	$9.66x10^{10}$
3		400	75	30000	3275	$0.98x10^8$		$9.38x10^6$	-1478	$6.55x10^{10}$
4		100	100	10000	3192	$3.2x10^7$	1797	$5.56x10^6$	-1395	$1.95x10^{10}$
5		400	2925	$1.17x10^6$	1763	$2.062x10^9$		$8.34x10^{11}$	34	$1.35x10^9$
6		250	250	62500	383	$2.4x10^7$		$2.17x10^8$	1414	$1.25x10^{11}$
7		900	300	270000	150	$4.1x10^7$		$2.03x10^9$	1647	$7.32x10^{11}$
Total				$\sum 1.868 x10^6$		$\sum 3.36 x10^9$		$\sum 8.37 x10^{11}$		$\sum 1.76x10^{12}$

Here,

Y_b	=	1.797 m
Y_t	=	1.703 m
Area	=	1.868 m^2
MOI_{girder}, I_c	=	2.597 m^4
Section $Modulus_b$, Z_b	=	1.445 m^3
Section $Modulus_t$, Z_t	=	1.525 m^3
Kern $Point_t$, K_t	=	0.774 m
Kern $Point_b$, K_b	=	0.817 m

Composite Section

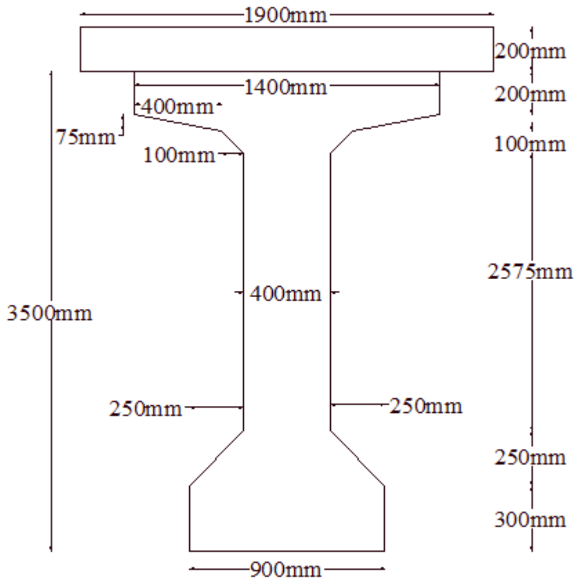


Figure-4 Composite Section at Middle

Modular Ratio, MOE of slab/girder = 0.94
Effective Flange Width = 1.9 m
Transformed Flange Width = 0.94x1.9 = 1.80 m
Transformed Flange Area = 0.94x1.9 x .2 = 0.36 m^2

Table -2 Section Property of Composite

Part	**A**	**Y**	**Ay**	**Y_N**	**I**	**Y_b**	**Ay_b^2**
	m^2	m	m^3	m	m^4	m	m^4
Girder	1.868	1.797	3.356	2.087	2.597	0.29	0.157
Slab	0.358	3.6	1.288		1.194×10^{-3}	-1.513	0.8195
Total	∑2.226		∑4.64		∑2.598		∑ 0.9765

Here,

Y'_b	=	2.09 m
Y'_t	=	1.41 m
Y'_{ts}	=	1.61 m
Area	=	2.23 m^2
MOI_{girder}, I'_c	=	3.58 m^4
Section $Modulus_b$, Z'_b	=	1.71 m^3
Section $Modulus_t$, Z'_t	=	2.53 m^3
Section $Modulus_{ts}$, Z'_{ts}	=	2.22 m^3
Kern $Point_t$, K'_t	=	0.77 m
Kern $Point_b$, K'_b	=	1.14 m
Constant $Factor_t$, m_t	=	0.60 m
Constant $Factor_b$, m_b	=	0.84 m

Concrete Volume in PC girder

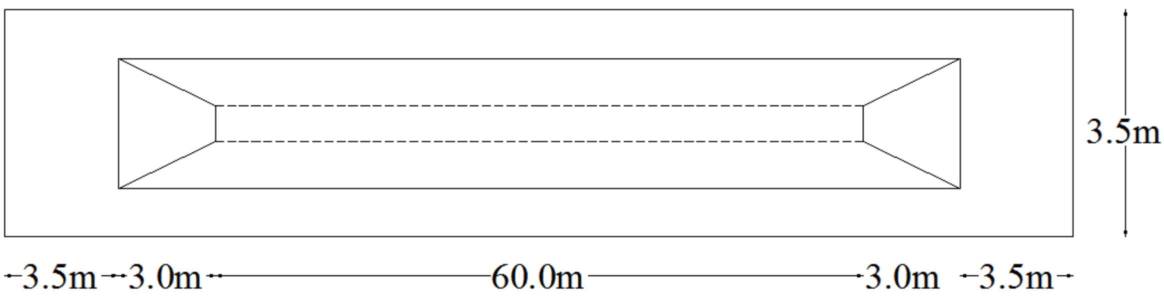


Figure-5 Elevation of PC Girder

Area of Mid-Block	=	1.87 m^2
Area of End Block	=	3.26 m^2
Area of Slopped Block	=	2.56 m^2
Total Volume of Girder	=	3.26x2x3.5+2.56x2x3+1.87x60
	=	150.27 m^3

Moment & Shear Calculation

<u>Calculation of Dead Load Moment</u>

a) Dead Load Moment due to Girder

Load from Mid-Block	=	1.87x24 = 44.82 KN/m
Load from End Block	=	3.26x24 = 78.28 KN/m
Load from Slopped Block	=	(44.82+78.28)/2 = 61.55 KN/m

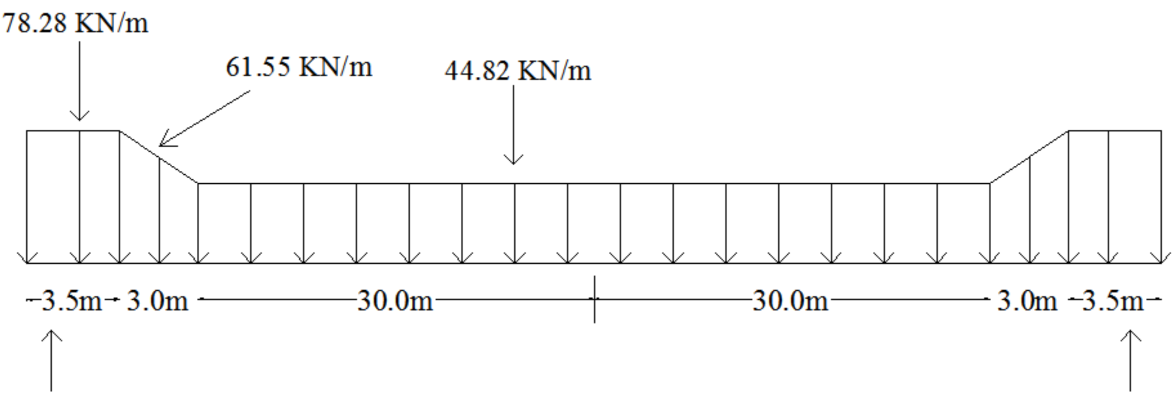


Figure-6 Self Weight of Girder

$$\text{Reaction at Support, } R_A = 78.28x3.5 + 61.55x3 + 44.82x30 = 1803.21 \text{ KN}$$

$$\text{Moment at Mid, } M_{L/2} = 1803.21x36.05 - (78.28x3.5x34.75) - (0.5x3x33.46x32) - (44.82x33x16.5) = 29475.02 \text{ KN} - \text{m}$$

b) Dead Load Moment due to Cross Girder

Load from Exterior = 3.2x1.5x0.6x24 = 69.12 KN

Load from Interior = 3.2x1.5x0.35x24 = 40.32 KN

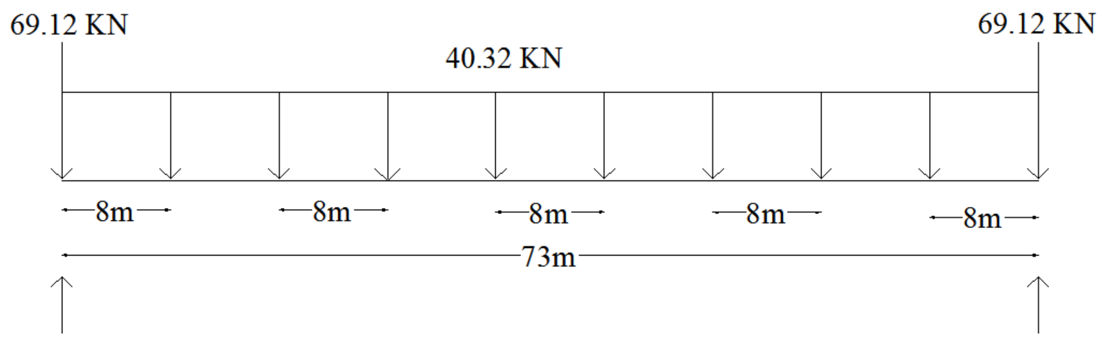


Figure-7 Load due to Cross Girder

$$\text{Reaction from Support, } R_A = \frac{(69.12 * 2 + 40.32 * 8)}{2} = 230.4 \text{ KN}$$

$$\text{Moment at Mid, } M_{\frac{L}{2}} = 230.4 * 36.05 - 69.12 * 36.05 - 40.32 * (28.05 + 20.05 + 12.05 \;\; + 4.05) = 3329.28 \text{ KN} - \text{m}$$

c) Dead Load Moment due to Deck Slab

Load from Deck, w = 1.9x0.2x24 = 9.12 KN/m

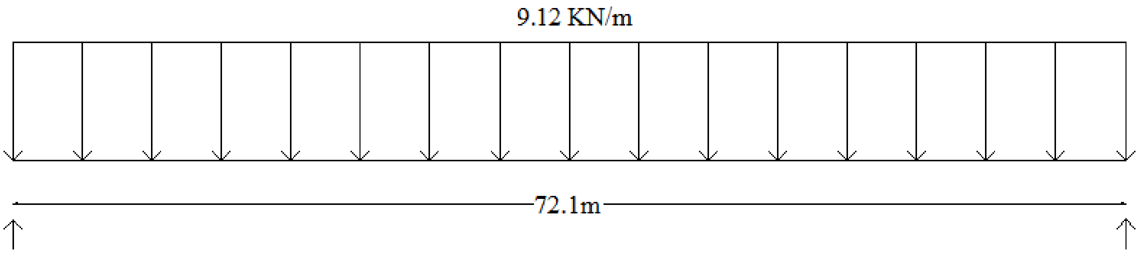


Figure -8 Self Weight of Deck Slab

Moment at Mid, $M_{L/2}$ = $wL^2/8 = (9.12x72.1^2)/8 = 5926.19$ KN-m

d) Dead Load Moment due to Wearing Course

Load from WC, w = 1.9x.075x23 = 3.42 KN/m

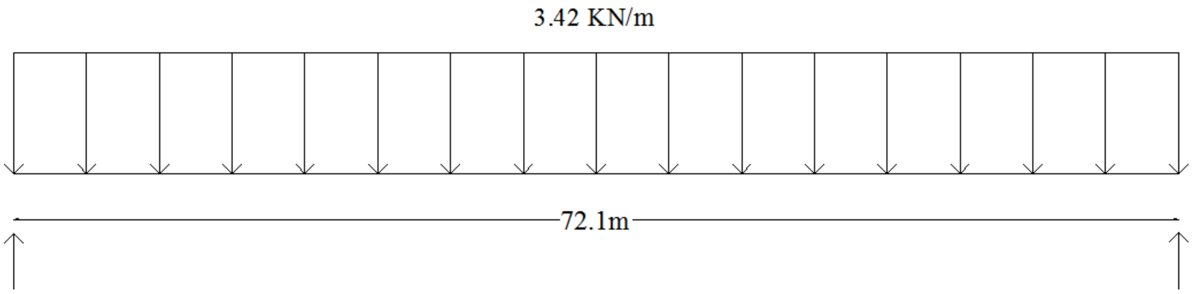


Figure -9 Self Weight of Wearing Course

Moment at Mid, $M_{L/2}$ = $wL^2/8 = (3.42x72.1^2)/8 = 2222.32$ KN-m

e) Total Dead Load Moment

M_{DL} = Girder + Cross Girder + Deck Slab + WC

= 29475.02+3329.28+5926.19+2222.32

= 40952.8116 KN-m

f) Total Factored Dead Load Moment

MF_{DL} = (29475.02+3329.28+5926.19)x1.25+2222.32x1.5

[AASTHO `07, Table 3.4.1-2]

= 51746.60 KN-m

Calculation of Live Load Moment

According to AASTHO LRFD HL 93 loading, each design lane should occupy either by the design truck or design tandem and lane load, which will be effective 3000mm transversely within a design lane. [AASTHO `07 3.6.1.2.1]

a) Distribution Factor for Moment

One Design Lane Loaded: $0.06+\left(\frac{S}{4300}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{Lt_s^3}\right)^{0.1}$ Two or More Design Lanes Loaded: $0.075+\left(\frac{S}{2900}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{Lt_s^3}\right)^{0.1}$	$1100 \le S \le 4900$ $110 \le t_s \le 300$ $6000 \le L \le 73\,000$ $N_b \ge 4$ $4\times10^9 \le K_g \le 3\times10^{12}$
use lesser of the values obtained from the equation above with $N_b = 3$ or the lever rule	$N_b = 3$

[AASTHO`07, Table 4.6.2.2.2b-1]

Here,

$$K_g = n\left(I + Ae_g^{\,2}\right) \qquad (4.6.2.2.1\text{–}1)$$

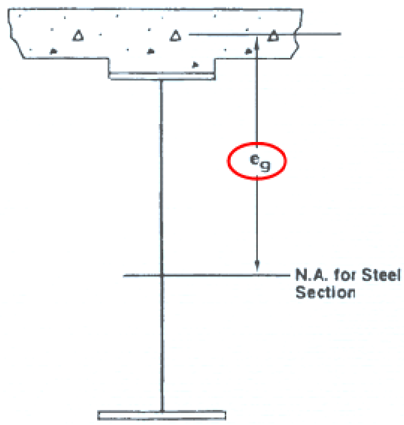
n = modular ratio

I = moment of inertia of steel girder

A = area of steel girder

e_g = distance between centers of gravity of steel girder and concrete deck

$\left(\frac{K_g}{Lt_s^3}\right)$ term may be taken as 1.0 for preliminary design



$$K_g = \frac{\sqrt{45}}{\sqrt{40}}\{3.58 + 2.23(1.61 - 0.1)^2\} = 9.20$$

$$DFM = 0.075 + \left(\frac{1.9}{2.9}\right)^{0.6} + \left(\frac{1.9}{72.1}\right)^{0.2}\left(\frac{9.20}{72.1*0.2^3}\right)^{0.1} = 0.57$$

b) Moment due to truck load

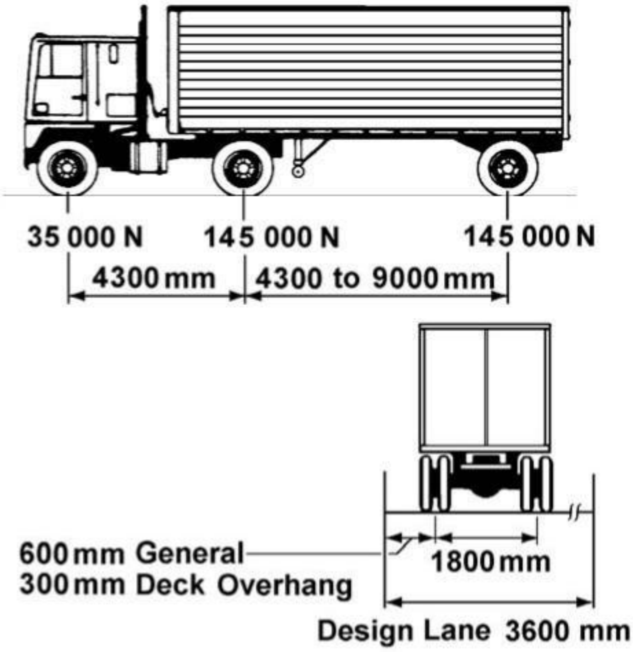


Figure-10 AASTHO HL-93 Truck Loading

Rear Wheel Load	= 145xDFM	= 145x0.57	= 82.59 KN
Front Wheel Load	= 35xDFM	= 35x0.57	= 19.93 KN

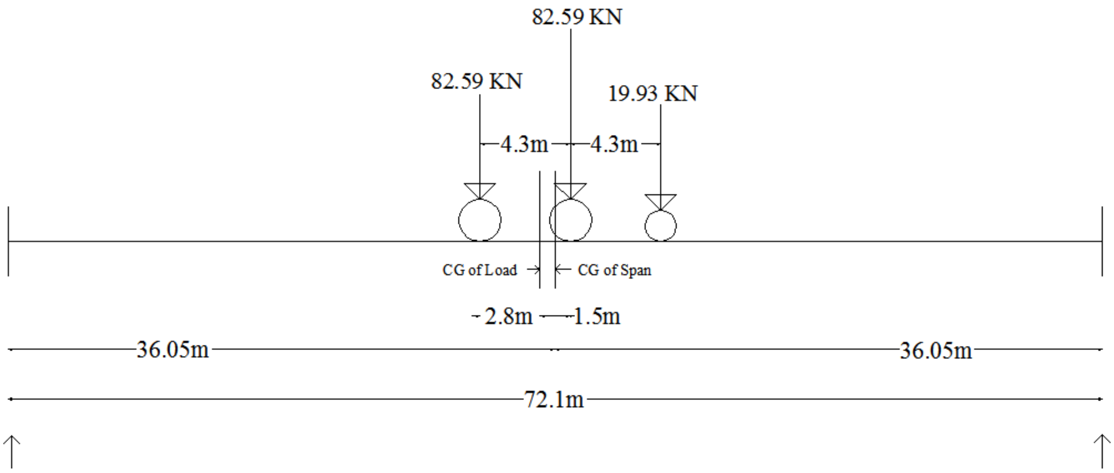


Figure-11 Truck Load

CG of Load $= \dfrac{82.59 * 0 + 82.59 * 4.3 + 19.93 * 8.6}{82.59 * 2 + 19.93} = 2.84 \text{ m}$

$$\text{Reaction, } R_A = \frac{82.59 * 39.62 + 82.59 * 35.32 + 19.93 * 31.02}{72.1} = 94.42 \text{ KN}$$

Moment at Mid, $M_{L/2}$ = 94.42x36.05-82.59(4.3-0.73) = 3108.83 KN-m

Impact Moment = 3108.83x0.33 = 1025.91 KN-m

[AASTHO`07, Table 3.6.2.1-1]

Total Live Load Moment due to Truck Load = 3108.83+1025.91 = 4134.74 KN-m

c) Moment due to tandem load [AASTHO`07, 3.6.1.2.3]

Wheel Load = 110x0.57 = 62.65 KN

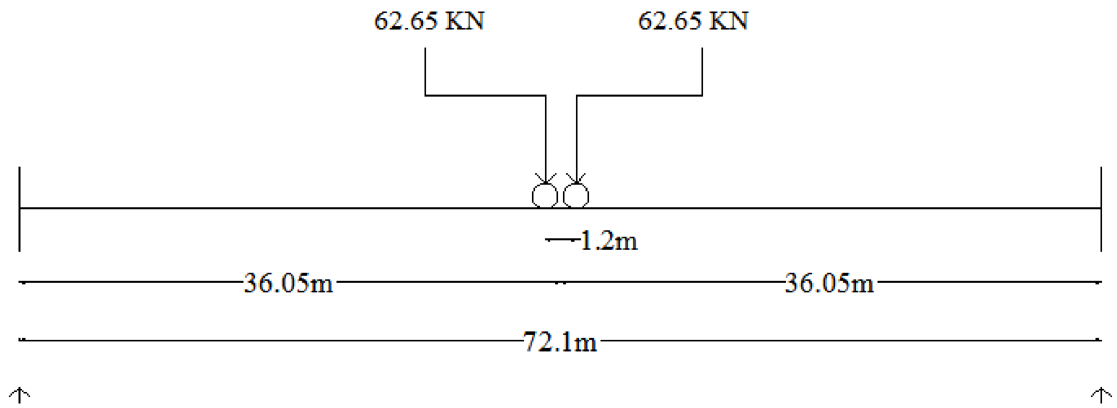


Figure-12 AASTHO Standard Tandem Loading

Moment at Mid, $M_{L/2}$ = 62.65x36.05-62.65*0.6 = 2220.94 KN-m

Impact Moment = 2220.94x0.33 = 732.91 KN-m

Total Live Load Moment due to Tandem Load = 2220.94+732.91 = 2953.85 KN-m

d) Moment due to lane load [AASTHO`07, 3.6.1.2.4]

Lane Load, w = 9.3xDFM = 9.3x.57 = 5.30 KN/m

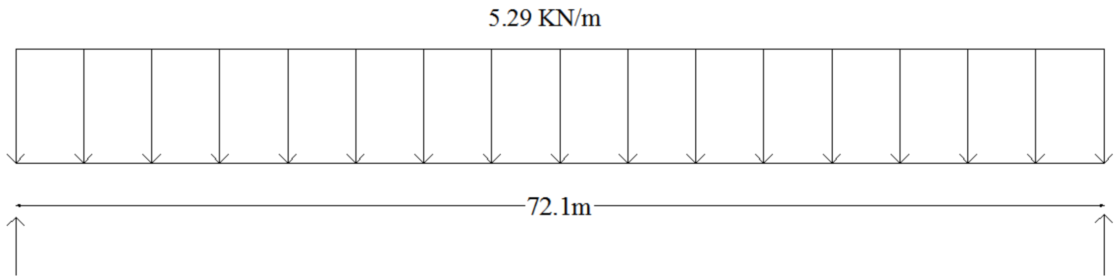


Figure -13 Lane Load

Moment at Mid, $M_{L/2}$ = $wL^2/8$ = $(5.30x72.1^2)/8$ = 3441.9 KN-m

Total Live Load moment

As Truck Load Moment is higher than Tandem Load Moment, the total vehicular live load moment as stated in AASTHO,

M_{LL} = Truck Load + Lane Load

= 4134.74+3441.87 = 7576.61 KN-m

Total Factored Live Load moment

M_{FLL} = 7576.61x1.75 = 13259.07 KN-m

[AASTHO`07, Table 3.4.1-1]

Shear Calculation

a) Distribution factor for Shear [AASTHO`07, Table 4.6.2.2.3a-1]

$$0.2+\frac{S}{3600}-\left(\frac{S}{10\ 700}\right)^{2.0}$$

$1100 \le S \le 4900$
$6000 \le L \le 73\ 000$
$110 \le t_s \le 300$
$N_b \ge 4$

$$\text{DFV} = 0.2 + \frac{1.9}{3.6} - \left(\frac{1.9}{10.7}\right)^2 = 0.70$$

b) Shear due to Dead Load

Shear due to Self-Weight of Girder	= 1803.22 KN
Shear due to Cross Girder	= 203.4 KN
Shear due to Deck Slab	= 9.12x (72.1/2) KN = 328.78 KN
Shear due to Wearing Course	= 3.42x (72.1/2) KN = 123.29 KN
Total Dead Load Shear	= 2458.70 KN

c) Shear due to Live Load

Shear due to Truck Load	= 94.42 KN
Impact Shear	= (94.42x0.33) = 31.16 KN

Total Shear due to Truck = (94.42+31.1586) KN = 125.58 KN

Load due to lane per unit Length = 9.3x0.70 KN/m =6.47 KN/m

Shear Due to Lane Load = (6.47x72.1)/2 KN = 233.34 KN

Total Live Load Shear = (125.58+233.34) KN = 358.92 KN

Total Shear, V_{D+L} = 2458.70+358.92 = 2817.62 KN

Total Factored Shear, $V_{F(D+L)}$ = (2458.70x1.5+358.92x1.75) = 4316.14 KN

Estimation of Required Pre-stressed Force and Number of Cable

Assumed Number of Cable = 9

CG of the Cable at Girder Mid = 503.33 mm

Eccentricity at Mid-Section = (1.79-0.503) m = 1293.7 mm

$$\text{Required Prestress Force,} \quad F = \frac{M_p + M_C * mb - f'_b * k_t * A_c}{e + k_t}$$

For full prestressing, $f'_b = 0$ [Design of Prestressed Concrete Structures, T.Y. Lin, Chapter 6, Equation 6-18]

Here,

M_P= Moment due to Girder, Cross Girder & Deck Slab [Precast]

M_C = Moment due to Live Load & Wearing Course [Composite, Service Condition]

$$= \frac{38730.49 + 9798.93 * 0.84}{1.29 + 0.77} = 22797 \text{ KN}$$

$$\text{Required Steel Area, } A_s = \frac{22797 * 1000}{0.6 * 1860} = 20403 \text{ mm}^2$$

Using 19T15 Strand, Required Cable = (20403.18/2660)

= 7.67 Nos.

Here, Pre-stressing is done in two stages

No of Cable at Stage I = 7

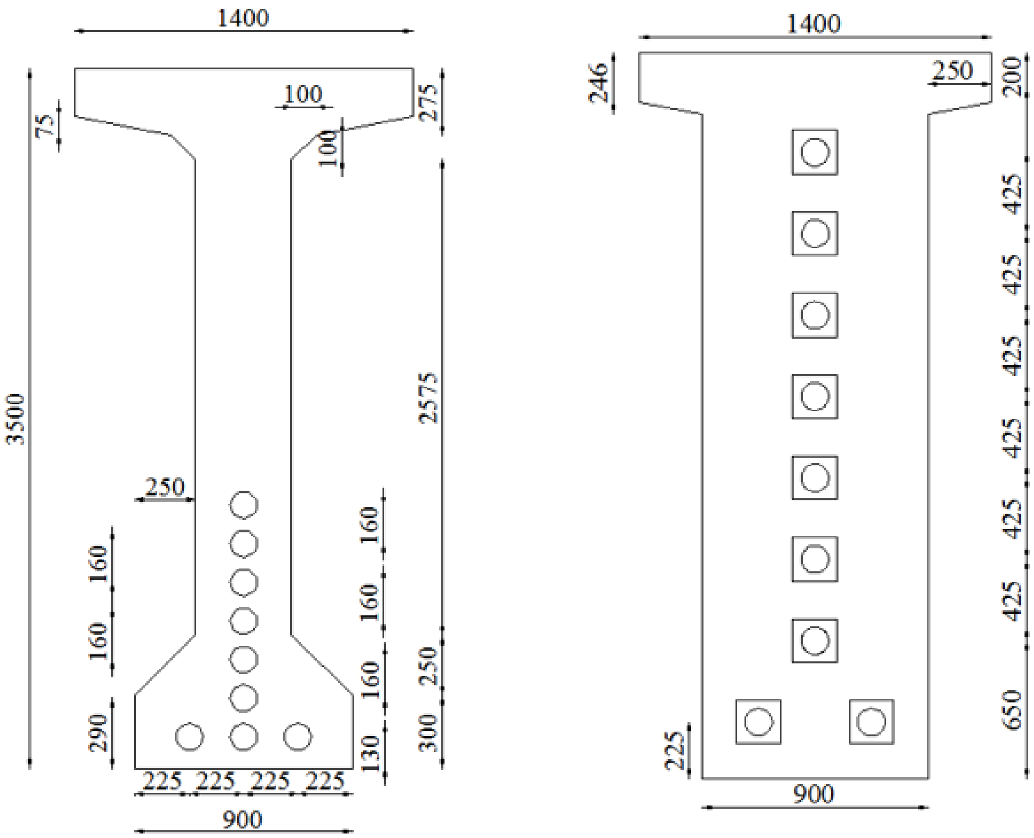
No of Cable at Stage II	= 2
Jacking Force at Stage I	= 3710x7 = 25970 KN
Jacking Force at Stage II	= 3710x2 = 7420 KN
Total jacking force (I+II)	= 33390 KN

<u>Stage-I</u>

CG of cable (7 Nos.) at Stage I	= 610 mm
Eccentricity of Girder Section	= 1186.82 mm
Eccentricity of Composite Section	= 1477.07 mm

<u>Stage-II</u>

CG of cable (2 Nos.) at Stage II	= 130 mm
Eccentricity of Girder Section	= 1666.82 mm
Eccentricity of Composite Section	= 1957.07 mm



a) Middle Section b) End Section

Figure-14 Cable arrangement in Mid and End Section of Girder

Calculation of Loss

Instantaneous Loss

a) Friction and Wedge Pull Loss

Table-3 Friction and Wedge Pull Loss

Cable No	Vertical Sag, d_r	Horizontal Sag	Radius of Curvature, R	α=X/R (X=1m)	α=X/R (X=L/2)
1	90	0	7220.014	0.000139	0.005035
2	90	0	7220.014	0.000139	0.005035
3	492	0	1320.734	0.000757	0.027523
4	743	0	874.5640	0.001143	0.041564
5	994	0	653.7240	0.001530	0.055605
6	1245	0	521.9290	0.001916	0.069646
7	1495	0	434.6500	0.002301	0.083631
8	1746	0	372.1660	0.002687	0.097672
9	1997	0	325.3890	0.003073	0.111713

Cable No	Initial Prestress Force, KN	Loss (per m), KN	Loss (L/2), KN	Wedge Pull Effect Distance, X_A	Loss due to Wedge Pull (if $X_A > L/2$	Friction + Wedge Pull Loss (%)
1	3710	2.58	92.55	34.92	0	2.495
2		2.58	92.55	34.92	0	2.495
3		3.16	113.07	31.53	0	3.048
4		3.53	125.79	29.85	0	3.391
5		3.89	138.45	28.42	0	3.731
6		4.25	151.05	27.18	0	4.071
7		4.62	163.58	26.09	0	4.409
8		4.98	173.03	25.13	0	4.664
9		5.34	188.41	24.27	0	5.078
∑ Force =	33390	∑ Loss =	1238.48	Percent Loss =		3.71

Friction Loss [Sample Calculation of Cable 1]

Friction Loss, $\Delta f_{pF} = \Delta f_{pi}\ [1 - e^{-(kx+\mu\alpha)}]$[AASTHO`07, Equation 5.9.5.2.2b-1]

Here,

Friction Co-Efficient, μ = 0.25 [AASTHO`07, Table 5.9.5.2.2b-1]

Wobble Co-Efficient, k = 0.00066 /m

$\alpha = \sqrt{\alpha_v{}^2 + \alpha_H{}^2} \qquad \alpha = \frac{X}{R}$

$$\text{Radius of Curvature, } R = \frac{L^2}{8 * dr} = \frac{72.1^2}{8 * 0.09} = 7220 \text{ rad}$$

$$dr = \text{vertical sag height} = 220 - 130 = 90 \text{ mm}$$

$$\alpha = \frac{X}{R} = \frac{1}{7220} = 1.39 * 10^{-4}; \quad X = 1 \text{ m}$$

$$\alpha = \frac{X}{R} = \frac{36.05}{7220} = 5.03 * 10^{-3}; \quad X = \frac{L}{2} = 36.05 \text{ m}$$

$$\text{Friction Loss, } \Delta f_{pF} = 3710\left[1 - e^{-\left(0.00066*1+0.25*1.39*10^{-4}\right)}\right] = 2.57 \text{ KN}; \ X = 1 \text{ m}$$

$$\text{Friction Loss, } \Delta f_{pF} = 3710\left[1 - e^{-\left(0.00066*36.05+0.25*5.03*10^{-3}\right)}\right] = 92.55 \text{ KN}; \ X = 36.05 \text{ m}$$

Anchorage Slip Loss [Calculation of Cable 1]

Let's Assume, Anchorage Slip = 6 mm [AASTHO`07, C5.9.5.2.1]

Distance until where anchorage slip loss will be effective,

$$X_A = \sqrt{\frac{\text{Slip} * E_p * A_s}{\Delta f_{pF}}} = \sqrt{\frac{0.006 * 197 * 2660}{2.57}} = 34.92$$

Anchorage Slip loss will only occur when $X_A > \frac{L}{2}$

$$\text{Loss due to slip} = 2 * \Delta f_{pF} * \left(X_A - \frac{L}{2}\right) = 0$$

Friction and Slip Loss = 92.55 + 0 = 92.55 KN

$$\text{Percentage (\%)} = \frac{92.55}{3710} * 100 = 2.495\%$$

b) Elastic Shortening Loss [AASTHO`07, 5.9.5.2.3b-1]

$$\Delta\, f_{PES} = \frac{N-1}{2N}\frac{Ep}{Eci}\, fcgp$$

$$fcgp = \frac{Peff}{A} + \frac{Peff * e\text{^}2}{I} - \frac{Mg * e}{I}$$

$$= \frac{3460 * 9}{1.87} + \frac{3460 * 9 * 1.29^2}{2.59} - \frac{29475 * 1.29}{2.59} = 22.05 \text{ MPa}$$

$$\Delta\, f_{PES} = \frac{9-1}{2 * 9} * \frac{197 * 10^3}{26290} * 22.05 = 73.43 \text{ MPa}$$

$$\text{Percent of Elastic Shortening Loss } = \frac{73.43 * 2660}{3710 * 1000} * 100 = 5.26\ \%$$

$$\text{Total Instantaneous Loss } = 3.71 + 5.26 = 8.97\ \%$$

Long Term / Time Dependent Loss

Approximate Estimate of Time-Dependent Losses, [AASTHO`07, 5.9.5.3-1]

Long Term Loss due to Shrinkage, Creep and Still Relaxation is given below.

$$\Delta f = 10 * \frac{f_{pi} * A_{ps}}{A_g} * \gamma_h \gamma_{st} + 83\gamma_h \gamma_{st} + \Delta f_{pR}$$

$$\gamma h = 1.7 - 0.01H, \qquad \text{Relative Humidity, } H = 70\%$$

$$\gamma_{st} = \frac{35}{7 + f'ci} = \frac{35}{7+30} = 0.946$$

$$\Delta\, f_{pLT} = \left[\frac{10 * 0.75 * 1860 * 2660 * 9}{2.226 * 10^6} * (1.7 - 0.01 * 70\%) * \left(\frac{35}{7+30}\right) + 83 * (1.7 - 0.01 * 70\%) * \left(\frac{35}{7+30}\right) + 17\right] = [141.92 + 78.51 + 17]$$
$$= 237.43 \text{ MPa}$$

$$\text{Percent of Time Dependent Loss } = \frac{237.43 * 2660}{3710 * 1000} * 100 = 17\%$$

Total Loss (Instantaneous & Time Dependent) = (17+8.97) % = 25.97%

Revised No of Required Cable

Total Percent of Loss = 25.97%

Total Loss = 362.28 MPa

Effective Steel Stress after Loss = 0.75*1860-362.28 =1043.88 MPa

Revised No of Required Cable,

$$\frac{\text{Requred Effective Force} * 1000}{A_{ps} * \text{Effective Steel Stress after Loss}} = \frac{22797 * 1000}{2660 * 1043.88} = 8.21 \text{ Nos.}$$

Actual Effective Force per Cable = 1043.88x (2660/1000) =2776.72 KN

Actual Effective Stress per Cable = 1043.88 MPa

Stress Calculation

Table-4 Calculation of Stress in different stages

Calculation of Stress	
Stress due to Self-Weight of Girder [+ = Compressive - = Tension]	$\sigma = \left(\frac{M_g * Y}{I_g}\right) * \frac{1}{1000}$ $\sigma_b = -\frac{29475 * 1.79}{2.59 * 1000} = -20.37 \text{ MPa}$ $\sigma_t = +\frac{29475 * 1.70}{2.59 * 1000} = +19.38 \text{ MPa}$
Stress due to PS-I Force	$\sigma = \left(+\frac{\text{Jacking Force}}{A_g} \pm \frac{\text{Jacking Force} * e}{Z_b}\right) * \frac{1}{1000}$ $\sigma_b = \left(+\frac{3710 * 7}{1.868} + \frac{3710 * 1.186 * 7}{1.445}\right) = 35.22 \text{ MPa}$ $\sigma_t = \left(+\frac{3710 * 7}{1.868} - \frac{3710 * 1.186 * 7}{1.525}\right) = -6.29 \text{ MPa}$
Stress due to Friction and Slip loss (3.71%)	$\sigma_b = -35.22 * 3.71\% = -1.3 \text{ MPa}$ $\sigma_t = -(-6.29) * 3.71\% = +0.24 \text{ MPa}$
Stress due to Elastic Shortening (5.23%)	$\sigma_b = -35.22 * 5.23\% = -1.84 \text{ MPa}$ $\sigma_t = -(-6.29) * 5.23\% = +0.33 \text{ MPa}$

Stress due to $\frac{1}{2}$ Time Dependent Loss of PS-I (8.5%)	$\sigma_b = -35.22 * 8.5\% = -2.9\ \text{MPa}$ $\sigma_t = -(-6.29) * 8.5\% = +0.53\ \text{MPa}$
Stress due to PS-II Force	$\sigma_b = \left(\frac{3710 * 2}{1.868} + \frac{3710 * 2 * 1.666}{1.445}\right) = 12.53\ \text{MPa}$ $\sigma_t = \left(\frac{3710 * 2}{1.868} - \frac{3710 * 1.196 * 2}{1.525}\right) = -1.847\ \text{MPa}$
Stress due to Friction and Slip Loss (3.71%)	$\sigma_b = -12.53 * 3.71\% = -0.46\ \text{MPa}$ $\sigma_t = -(-1.847) * 3.71\% = +0.0685\ \text{MPa}$
Stress due to Elastic Shortening (5.23%)	$\sigma_b = -12.53 * 5.23\% = -0.655\ \text{MPa}$ $\sigma_t = -(-1.847) * 5.23\% = +0.0965\ \text{MPa}$
Stress due to Self-Weight of Deck Slab	$\sigma_b = -\frac{5926.18 * 1.797}{2.5972 * 1000} = -4.1\ \text{MPa}$ $\sigma_t = +\frac{5926.18 * 1.703}{2.5972 * 1000} = +3.88\ \text{MPa}$
Stress due to Self-Weight of Cross Girder	$\sigma_b = -\frac{3329.28 * 1.797}{2.5972 * 1000} = -2.3\ \text{MPa}$ $\sigma_t = +\frac{3329.28 * 1.703}{2.5972 * 1000} = +2.18\ \text{MPa}$
Stress due to $\frac{1}{3}$ Time Dependent Loss of PS-II (5.67%)	$\sigma_b = -12.53 * 5.67\% = -0.71\ \text{MPa}$ $\sigma_t = -(-1.847) * 5.67\% = +0.105\ \text{MPa}$
Stress due to Other Half Time	$\sigma_b = -\left(\frac{3710 * 7 * 8.5\%}{2.226}\right) - \left(\frac{3710 * 7 * 8.5\% * 1.47}{1.71}\right)$ $= -2.90\ \text{MPa}$ $\sigma_t = -\left(\frac{3710 * 7 * 8.5\%}{2.226}\right) + \left(\frac{3710 * 7 * 8.5\% * 1.47}{2.53}\right)$

<table>
<tr><td>Dependent Loss of PS-I (8.5%)</td><td>$= +0.29 \text{ MPa}$
$\sigma_{st} = -\left(\frac{3710*7*8.5\%}{2.226}\right)+\left(\frac{3710*7*8.5\%*1.47}{2.22}\right)$
$= +0.47 \text{ MPa}$</td></tr>
<tr><td>Stress due to other $\frac{2}{3}$loss Time Dependent Loss of PS-II (11.33%)</td><td>$\sigma_b = -\left(\frac{3710*2*11.33\%}{2.226}\right)-\left(\frac{3710*2*11.33\%*1.47}{1.71}\right)$
$= -1.10 \text{ MPa}$

$\sigma_t = -\left(\frac{3710*2*11.33\%}{2.226}\right)+\left(\frac{3710*2*11.33\%*1.47}{2.53}\right)$
$= +0.11 \text{ MPa}$
$\sigma_{st} = -\left(\frac{3710*2*11.33\%}{2.226}\right)$
$+\left(\frac{3710*2*11.33\%*1.47}{2.22}\right)$
$= +0.18 \text{ Mpa}$</td></tr>
<tr><td>Stress Due To Differential Shrinkage of Deck Slab</td><td>Tensile Stress in-situ Slab,
$T = 1*10^{-4}*E_c\sqrt{\frac{f_s}{f_g}}*1000$
$\text{So}, T = 1*10^{-4}*32200*\sqrt{\frac{40}{45}} = 3.03 \text{ MPa}$

Compressive Force at CG of Slab, P = T x S x t_sx 1000
=3.03 x 1.9 x 0.2 x 1000 = 1153.62 KN

C G of Slab from Composite $Y_{t,}$ (Y_t- t_s/2) = (1.41-0.2/2)= 1.31 m

$\sigma_b = \frac{P}{A_c} - \frac{P*1.31}{Z_b'} = \frac{1153.62}{2.226} - \frac{1153.62*1.31}{1.71}$
$= -0.365 \text{ MPa}$
$\sigma_t = \frac{P}{A_c} + \frac{P*1.31}{Z_b'} = \frac{1208.4}{2.226} + \frac{1208.4*1.31}{2.53} = +1.12 \text{ MPa}$

σ_{st} = (Stress Girder Top Fiber - Tensile Stress in-situ Slab T)
= +1.12-3.03 = - 1.91 MPa</td></tr>
</table>

Stress due to Self-Weight of Wearing Course	$\sigma_b = -\frac{2222.32 * 2.087}{3.58 * 1000} = -1.29\ \text{MPa}$ $\sigma_t = +\frac{2222.32 * 1.413}{3.58 * 1000} = +0.56\ \text{MPa}$ $\sigma_{ts} = \frac{2222.32}{2.22 * 1000} = +1\ \text{MPa}$
Stress due to Design Live Load	$\sigma_b = -\frac{7576.61 * 2.087}{3.58 * 1000} = -4.41\ \text{MPa}$ $\sigma_t = \frac{7576.61 * 1.413}{3.58 * 1000} = +3\ \text{MPa}$ $\sigma_{st} = \frac{7576.61}{2.226 * 1000} = +3.397\ \text{MPa}$

Table-5 Schedule of Stress

	Case	Stage of Stress	F_{bottom}	F_{top}	$F_{slab\ top}$
R1	Effect due to Self-Weight of Girder and PS-I transfer after IL	Dead Load of Naked Girder	-20.37	+19.38	0
		PS-I transfer	+35.22	-6.29	0
		Instantaneous Loss (Friction+Slip+ES)	-3.14	+0.57	0
		Resultant of PS-I	+11.71	+13.66	0
		Permissible of PS-I	+18.00	-1.36	0
R2	Effect due to ½ of TDL	½ Time Dependent Loss of PS-I	-3.00	+0.53	0

		Resultant after ½ TDL of PS-I	+8.81	+14.19	0
R3	Effect due to PS-II transfer after IL	PS-II transfer	+12.53	-1.847	0
		Instantaneous Loss	-1.115	+0.165	0
		Resultant of PS-II	+20.225	+12.50	0
		Permissible of PS-II	24	-1.57	0
R4	Effect due to self-weight of Deck Slab and Cross Girder	Dead Load of Deck Slab	-4.10	+3.88	0
		Dead Load of Cross Girder	-2.3	+2.18	0
		Resultant Stress	+13.825	+18.56	0
R5	Effect due to 1/3 TDL Loss of PS-II	$\frac{1}{3}$ Time Dependent Loss of PS-II	-0.71	+1.05	0
		Resultant Stress	+13.115	+19.61	0
R6	Effect due to ½ and 2/3 TDL of PS-I and PS-II	½ Time Dependent Loss of PS-I (Composite)	-2.90	+0.29	+0.47
		$\frac{2}{3}$ Time Dependent Loss of PS-II (Composite)	-1.11	+0.11	+0.18
		Resultant Stress	+9.11	+20.01	+0.65

R7	Effect due to Self-Weight of Wearing Course and Stress due to DS	Stress due to Differential Shrinkage	-0.365	+1.12	-1.91
		Wearing Course on Composite Section	-1.9	+0.56	+1
		Resultant on Composite	+6.854	+21.69	-0.26
		Permissible Stress	-3.34	+22.5	+18
R8	Effect due to Live Load for Service I	Stress due to Design Live Load	-4.40	+3	+3.39
		Resultant Stress, Service I (Total DL+PS+Live Load)	+2.454	+24.69	+3.13
		Permissible Stress at Service I	-3.34	27	24
R9	Effect due to Live Load for Service III	Resultant Stress, Service III (Total DL+PS+0.8Live Load)	+3.33	+24.09	+2.45
		Permissible Stress at Service III	-3.34	27	24

Checking of Moment Capacity

Factored Moment (DL+LL) = (13259.07+61429.22) = 74688.28 KN-m

$$k = 2\left(1.04 - \frac{f_{pu}}{f_{py}}\right) = 2(1.04 - 0.9) = 0.28$$

$$d_p = 3500 + 200 - 503.33 = 3196mm$$

Let's Assume Rectangular Behavior,

$$c = \frac{(A_{ps} f_{pu} + A_s f_y - A_s{}' f_y{}'}{\left(0.85f'_c \beta_1 b + \frac{kA_{ps} f_{pu}}{d_p}\right)}$$

$$= \frac{2660 * 9 * 1860}{\left(0.85 * 40 * 0.76 * 1900 + \frac{0.28 * 2660 * 1860 * 9}{3196}\right)} = 840 > 200, T\ action$$

Let's Assume T behavior,

$$c = \frac{(A_{ps} f_{pu} + A_s f_s - A_s' f'_s - 0.85f'_c (b - b_w) h_f}{\left(0.85f'_c \beta_1 b_w + \frac{kA_{ps} f_{pu}}{d_p}\right)}$$

$$= \frac{2660 * 9 * 1860 - 0.85 * 45\ (1900 - 400) * 200}{\left(0.85 * 40 * 0.76 * 400 + \frac{0.28 * 2660 * 1860 * 9}{3196}\right)} = 2.32m$$

$$\alpha = \beta_1 * c = 0.76 * 2.35 = 1.76\ m$$

$$f_{ps} = f_{pu}\left(1 - \frac{k * c}{d_p}\right) = 1860\left(1 - \frac{0.28 * 2.32}{3.196}\right) = 1480\ MPa$$

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_s\left(d_s - \frac{a}{2}\right) + A_s{}' f_y{}'\left(d'_s - \frac{a}{2}\right)$$
$$+ 0.85f'_c (b - b_w) h_f\left(\frac{a}{2} - \frac{h_f}{2}\right)$$

$$M_n = 2660 * 9 * 1480 * \left(3196 - \frac{1760}{2}\right) + 0.85 * 40 * (1900 - 400) * 200 * \left(\frac{1760}{2} - \frac{200}{2}\right) = 90015 \text{ KN} - \text{m}$$

$$M_r = \emptyset * M_n = 0.9 * 90015 = 81015 \text{ KN} - \text{m}$$

$$M_u = 74688 \text{ KN} - \text{m}$$

$$M_r > M_u \text{ (ok)}$$

Deflection Calculation

CG of cable (7 Nos.) at PS-I = 610 mm

CG of cable (2 Nos.) at PS-II = 130 mm

CG of Cable at Girder End at PS-I = 1925 mm

CG of Cable at Girder End at PS-II = 225 msm

a) **Deflection due to self-weight of girder**

$$\text{Self} - \text{weight w} = \frac{\text{volumn} * \text{ unit weight}}{\text{span}} = \frac{150.2675 \text{ x}24}{73} = 49.40 \text{ KN/m}$$

$$\text{Deflection} = \frac{5\text{wL}^4}{384 \text{ x Eci x Ic}} = \frac{5 \text{ x } 49.40 \text{ x}73^4}{384 \text{ x } 26290 \text{ x } 2.59} = 268 \text{ mm}$$

b) **Deflection due to PS-I**

CG of Cable at Girder End = 1925 mm

Sag Height = (1925 – 610) mm = 1315 mm

Eccentricity of End Section = (1797-1925) mm = -118 mm

$$\text{Average Prestressed Force after IL} = 3710 \text{ x } 7\left(1 - \frac{5.23}{100} - \frac{3.71}{100}\right) = 23648 \text{ KN}$$

$$\text{Equivalent upward UDL due to Cable Parabola} = \frac{8 \text{ x p x sag}}{L^2} = \frac{8 \text{ x } 23648 \text{ x } 1.315}{73^2}$$

$$= 46.68 \text{ KN/m}$$

Upward deflection due to PS-I transfer,

$$\frac{5wL^4}{384 \text{ x Eci x I}} = \frac{5 \text{ x } 46.68 \text{ x } 73^4}{384 \text{ x } 26290 \text{ x } 2.59} = 253.5 \text{ mm}$$

Deflection at Girder End eccentricity,

$$\frac{\text{p x } e_{\text{end x}} L^2}{8 \text{ x Eci x I}} = \frac{23648 \text{ x } (-0.118) \text{ x } 73^2}{8 \text{ x } 26290 \text{ x } 2.59} = -27.3 \text{ mm}$$

c) Deflection due to PS-II

CG of Cable in PS-II at girder end = 225 mm

Sag Height = (225 – 130) = 95 mm

Eccentricity at PS-II cable at girder end = (1.797-0.225) =1.572 m

$$\text{Average Prestressed Force after IL} = 3710 \text{ x } 2\left(1 - \frac{5.23}{100} - \frac{3.71}{100}\right) = 6756.7 \text{ KN}$$

$$\text{Equivalent upward UDL due to Cable Parabola} = \frac{8 \text{ x p x sag}}{L^2} = \frac{8 \text{ x } 6756.7 \text{ x } 0.095}{73^2}$$

$$= 0.96 \text{ KN/m}$$

Upward deflection due to PS-II transfer,

$$\frac{5wL^4}{384 \text{ x Eci x I}} = \frac{5 \text{ x } 0.96 \text{ x } 73^4}{384 \text{ x } 30358 \text{ x } 2.59} = 4.5 \text{ mm}$$

Deflection at Girder End eccentricity,

$$\frac{\text{p x } e_{\text{end x}} L^2}{8 \text{ x Eci x I}} = \frac{6756.7 \text{ x } 1.572 \text{ x } 73^2}{8 \text{ x } 30358 \text{ x } 2.59} = 90.3 \text{ mm}$$

d) Net Deflection = (Net Hogging – Net Sagging)

$= [D_{PS\text{-}I} + D_{EPS\text{-}I} + D_{PS\text{-}II} + D_{EPS\text{-}II} – D_W]$

$=[253.5 – 27.3 + 4.5 + 90.3 – 268]$

= 53 mm

3. DESIGN EXAMPLE OF A TWO-WAY POST-TENSIONED SLAB

The following example illustrates the design methods presented in ACI 318-05 and IBC 2003. Unless otherwise noted, all referenced table, figure, and equation numbers are from these books. The example presented here is for Two-Way Post-Tensioned Design.

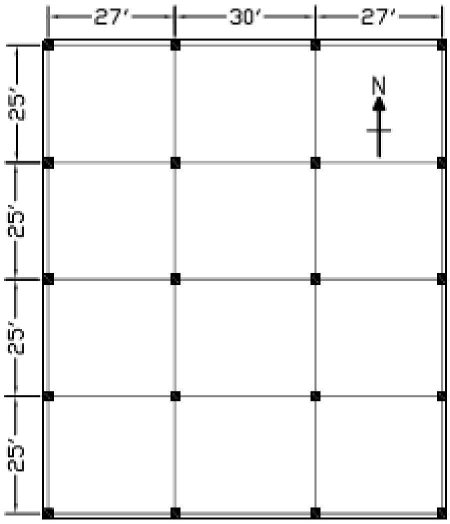


Figure 1 : Typical Plan of a Slab

Loads:

Framing Dead Load = self-weight
Superimposed Dead Load = 25 psf partitions, M/E, misc.
Live Load = 40 psf
residential 2 hour fire-rating
Materials:

Concrete:

Normal weight 150 pcf
f'_c = 5,000 psi
f'_{ci} = 3,000 psi

Rebar:

f_y = 60,000 psi
PT: Unbonded tendons 1/2"ϕ, 7-wire strands, A = 0.153 in^2
f_{pu} = 270 ksi
Estimated prestress losses = 15 ksi (ACI 18.6)
f_{se} = 0.7 (270 ksi) - 15 ksi = 174 ksi (ACI 18.5.1)
P_{eff} = A*fse = (0.153)(174 ksi) = 26.6 kips/tendon

Determine Preliminary

Slab Thickness

Start with L/h = 45
Longest span = 30 ft h = (30 ft)(12)/45 = 8.0" preliminary slab thickness

Loading

DL = Selfweight = (8in)(150 pcf) = 100 psf
SIDL = 25 psf
LLo = 40 psf

Design of East-West Interior Frame

Use Equivalent Frame Method, ACI 13.7 (excluding sections 13.7.7.4-5)
Total bay width between centerlines = 25 ft
Ignore column stiffness in equations for simplicity of hand calculations
No pattern loading required, since LL/DL < 3/4 (ACI 13.7.6)

Calculate Section Properties

Two-way slab must be designed as Class U (ACI 18.3.3),
Gross cross-sectional properties allowed (ACI 18.3.4)
$A = bh = (300\ in)(8\ in) = 2{,}400\ in^2$
$S = bh^2/6 = (300\ in)(8\ in)^2/6 = 3{,}200\ in^3$

Set Design Parameters Allowable stresses:

Class U (ACI 18.3.3) At time of jacking (ACI 18.4.1) $f'_{ci} = 3{,}000$ psi
Compression = $0.60\ f'_{ci} = 0.6(3{,}000\ psi) = 1{,}800$ psi
Tension = $3\sqrt{f'_{ci}} = 3\sqrt{3{,}000} = 164$ psi
At service loads (ACI 18.4.2(a) and 18.3.3)
$f'_c = 5{,}000$ psi
Compression = $0.45\ f'_c = 0.45(5{,}000\ psi) = 2{,}250$ psi
Tension = $6\sqrt{f'_c} = 6\sqrt{5{,}000} = 424$ psi

Average precompression limits:

P/A = 125 psi min. (ACI 18.12.4)
300 psi max.

Target load balances:

60%-80% of DL (selfweight) for slabs (good approximation for hand calculation)
For this example: $0.75\ W_{DL} = 0.75(100\ psf) = 75$ psf
Cover Requirements (2-hour fire rating, assume carbonate aggregate)
IBC 2003
Restrained slabs = 3/4″ bottom
Unrestrained slabs = 11/2″ bottom
= 3/4″ top

Tendon profile:

Parabolic shape; For a layout with spans of similar length, the tendons will be typically be located at the highest allowable point at the interior columns, the lowest possible point at the mid-spans, and the neutral axis at the anchor locations. This provides the maximum drape for load-balancing.

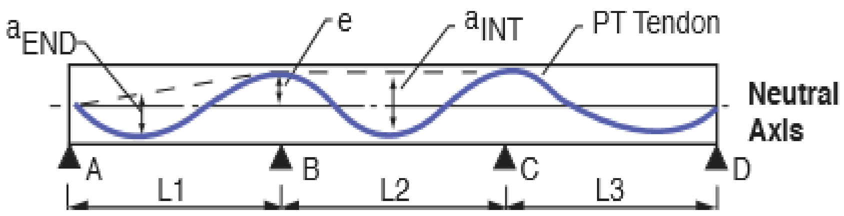


Figure 2: Tendon Profile

Tendon Ordinate	Tendon (CG) Location*
Exterior support – anchor	4.0”
Interior support – top	7.0”
Interior span – bottom	1.0”
End span – bottom	1.75”

(CG) = center of gravity
*Measure from bottom of slab

a_{INT} = 7.0” - 1.0” = 6.0”
a_{END} = (4.0” + 7.0”)/2 - 1.75” = 3.75”
eccentricity, e, is the distance from the center to tendon to the neutral axis; varies along the span

<u>Prestress Force Required to Balance 75% of selfweight DL</u>

Since the spans are of similar length, the end span will typically govern the maximum required post-tensioning force. This is due to the significantly reduced tendon drape, a_{END}.

$W_b = 0.75\ W_{DL}$
= 0.75 (100 psf)(25 ft)
= 1,875 plf
= 1.875 k/ft

For Exterior Span

Force needed in tendons to counteract the load in the end bay:

$P = W_b L^2 / 8a_{end}$
= (1.875 k/ft)(27 ft)^2 / [8(3.75 in / 12)]
= 547 k

Check Pre-compression Allowance

Determine number of tendons to achieve 547 k

tendons = (547 k) / (26.6 k/tendon)
= 20.56
Use 20 tendons

Actual force for banded tendons

P_{actual} = (20 tendons) (26.6 k) = 532 k

The balanced load for the end span is slightly adjusted w_b = (532/547)(1.875 k/ft) = 1.82 k/ft

Determine actual Precompression stress

P_{actual} /A = (532 k)(1000) / (2,400 in^2)

= 221 psi > 125 psi min. ok

< 300 psi max. ok

Check Interior Span Force

P = (1.875 k/ft)(30 ft)2 / [8(6.0 in / 12)]

= 421 k < 532 k Less force is required in the center bay

For this example, continue the force required for the end spans into the interior span and check the amount of load that will be balanced:

w_b = (532 k)(8)(6.0 in /12) / $(30\ ft)^2$

= 2.36 k/ft wb/wDL = 94%; [W_{DL}= 100*25=2.5 ksf]

This value is less than 100%; acceptable for this design.

East-West interior frame:

Effective prestress force, P_{eff} = 532 kips

Check Slab Stresses

Separately calculate the maximum positive and negative moments in the frame for the dead, live, and balancing loads. A combination of these values will determine the slab stresses at the time of stressing and at service loads.

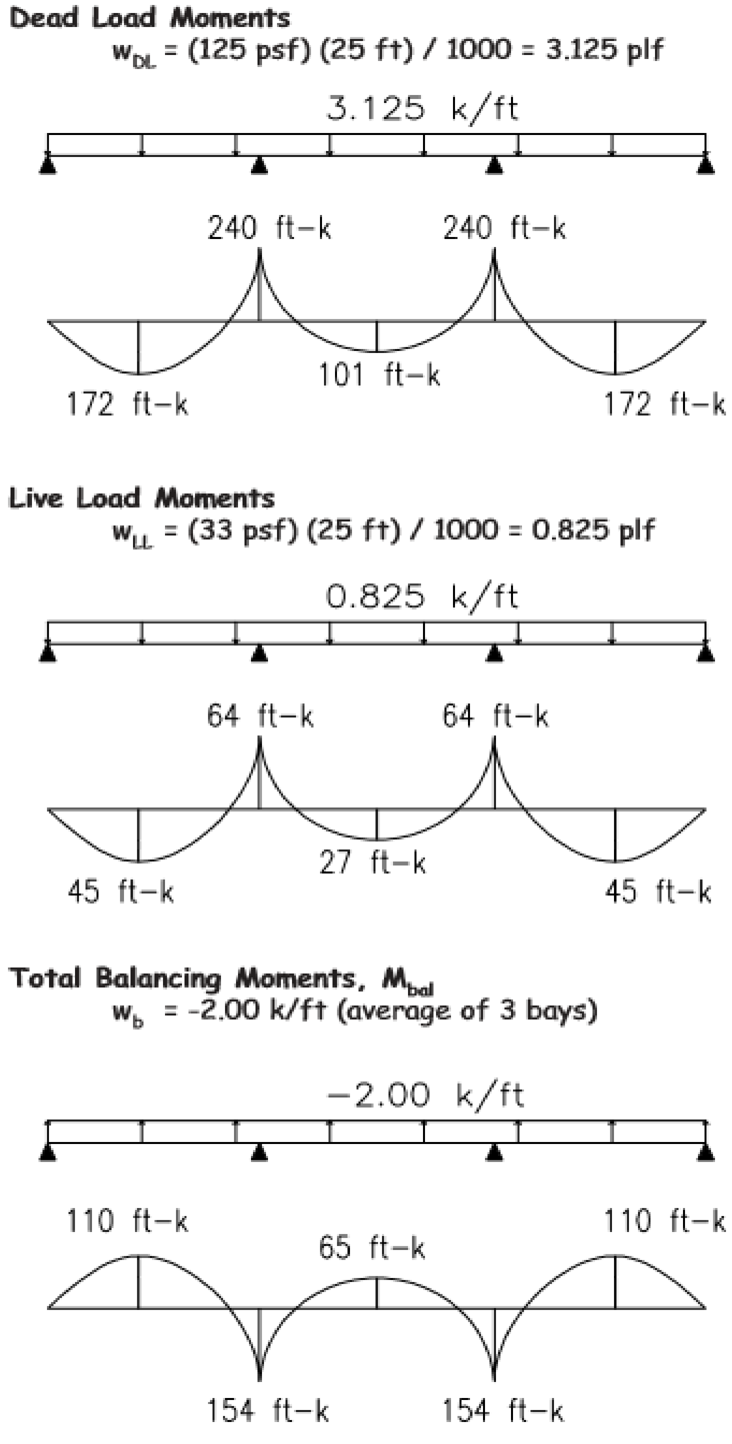


Figure 3: Moment Diagram for DL, LL and Balancing Load

Stage 1: Stresses immediately after jacking (DL + PT) (ACI 18.4.1)

Midspan Stresses

$f_{top} = (-M_{DL} + M_b)/S - P/A$

$f_{bot} = (+M_{DL} - M_b)/S - P/A$

Interior Span

$f_{top} = [(-101\text{ft-k} + 65\text{ft-k})(12)(1000)]/(3200\text{ in}^3) - 221\text{psi}$

$= -135 - 221 = -356$ psi compression < $0.60\ f'_{ci} = 1800$ psi ok

$f_{bot} = [(101\text{ft-k} - 65\text{ft-k})(12)(1000)]/(3200\text{ in}^3) - 221\text{psi}$

$= 135 - 221 = -86$ psi compression < $0.60\ f'_{ci} = 1800$ psi ok

End Span

$f_{top} = [(-172\text{ft-k} + 110\text{ft-k})(12)(1000)]/(3200\text{ in}^3) - 221\text{psi}$

$= -232 - 221 = -453$ psi compression < $0.60\ f'_{ci} = 1800$ psi ok

$f_{bot} = [(172\text{ft-k} - 110\text{ft-k})(12)(1000)]/(3200\text{ in}^3) - 221\text{psi}$

$= 232 - 221 = 11$ psi tension < $3\sqrt{f_{ci}} = 164$ psi ok

Support Stresses

$f_{top} = (+M_{DL} - M_b)/S - P/A$

$f_{bot} = (-M_{DL} + M_b)/S - P/A$

$f_{top} = [(240\text{ft-k} - 154\text{ft-k})(12)(1000)]/(3200\text{ in}^3) - 221\text{psi}$

$= 323 - 221 = 102$ psi tension < $3\sqrt{f'_{ci}} = 164$ psi ok

$f_{bot} = [(-240\text{ft-k} + 154\text{ft-k})(12)(1000)]/(3200\text{ in}^3) - 221\text{psi}$

$= -323 - 221 = -544$ psi compression < 0.60 f'ci = 1800 psi ok

Stage 2: Stresses at service load (DL + LL + PT) (18.3.3 and 18.4.2)

Midspan Stresses

$f_{top} = (-M_{DL} - M_{LL} + M_b)/S - P/A$

$f_{bot} = (+M_{DL} + M_{LL} - M_b)/S - P/A$

Interior Span

$f_{top} = [(-101\text{ft-k} - 27\text{ft-k} + 65\text{ft-k})(1000)]/(3200\text{ in}^3) - 221\text{psi}$

$= -236 - 221 = -457$ psi compression < $0.45\ f'_c = 2250$ psi ok

$f_{bot} = [(101\text{ft-k} + 27\text{ft-k} - 65\text{ft-k})(1000)]/(3200\text{ in}^3) - 221\text{psi}$

$= 236 - 221 = 15$ psi tension < $6\sqrt{f'_c} = 424$ psi ok

End Span

$f_{top} = [(-172\text{ft-k} - 45\text{ft-k} + 110\text{ft-k})(12)(1000)]/(3200\text{ in}^3) - 221\text{psi}$

$= -401 - 221 = -622$ psi compression < $0.45\ f'_c = 2250$ psi ok

$f_{bot} = [(172\text{ft-k} + 45\text{ft-k} - 110\text{ft-k})(12)(1000)]/(3200\text{ in}^3) - 221\text{psi}$

$= 401 - 221 = 180$ psi tension < $6\sqrt{f'_c} = 424$ psi ok

Support Stresses

$f_{top} = (+M_{DL} + M_{LL} - M_b)/S - P/A$

$f_{bot} = (-M_{DL} - M_{LL} + M_b)/S - P/A$

f_{top} = [(240ft-k + 64ft-k - 154ft-k)(12)(1000)]/(3200 in^3) - 221psi
= 563 - 221 = 342 psi tension < 6√f'$_c$ = 424 psi ok
f_{bot} = [(-240ft-k - 64 ft-k + 154ft-k)(12)(1000)]/(3200 in^3) - 221psi
= -563 - 221 = -784 psi compression < 0.45 f'$_c$ = 2250 psi ok

All stresses are within the permissible code limits.

Ultimate Strength

Determine factored moments
The primary post-tensioning moments, M_1, vary along the length of the span.
M_1 = P * e
e = 0 in. at the exterior support
e = 3.0 in at the interior support (neutral axis to the center of tendon)
M_1 = (532k)(3.0in) / (12) = 133ft-k

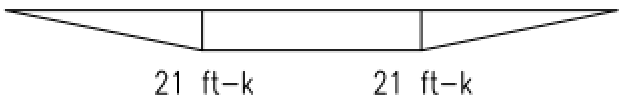


Figure 4: Secondary Moment Diagram

The secondary post-tensioning moments, M_{sec}, vary linearly between supports.
M_{sec} = M_b - M_1 = 154 ft-k - 133 ft-k
= 21 ft-k at the interior supports
The typical load combination for ultimate strength design is
M_u = 1.2 M_{DL} + 1.6 M_{LL} + 1.0 M_{sec}
At midspan:
M_u = 1.2 (172ft-k) + 1.6 (45ft-k) + 1.0 (10.5 ft-k) = 289 ft-k
At support:
M_u = 1.2 (-240ft-k) + 1.6 (-64ft-k) + 1.0 (21 ft-k) = -370 ft-k

<u>Determine minimum bonded reinforcement:</u>

To see if acceptable for ultimate strength design.

Positive moment region:

Interior span: f_t = 15 psi < 2√f'c = 2√5,000 = 141 psi
No positive reinforcement required (ACI 18.9.3.1)
Exterior span: f_t = 180 psi > 2√f'c = 2√5,000 = 141 psi

Minimum positive moment reinforcement required (ACI 18.9.3.2)
y = $f_t/(f_t + f_c)h$
= [(180)/(180+622)](8 in)
= 1.80 in
N_c = M_{DL+LL}/S * 0.5 * y * l2
= [(172 ft-k + 45 ft-k)(12) / (3,200 in^3)](0.5)(1.80 in)(25ft)(12) = 220 k

$A_{s,\ min} = N_c / 0.5fy$ = (220 k) / [0.5(60ksi)] = 7.33 in^2

Distribute the positive moment reinforcement uniformly across the slab-beam width and as close as practicable to the extreme tension fiber.

$A_{s,\ min}$ = (7.33 in2)/(25 ft) = 0.293in^2/ft

Use #5 @ 12 in. oc Bottom = 0.31 in^2/ft (or equivalent)

Minimum length shall be 1/3 clear span and centered in positive moment region (ACI 18.9.4.1)

Negative moment region:

$A_{s,\ min} = 0.00075A_{cf}$ (ACI 18.9.3.3)

Interior supports:

A_{cf} = max. (8in)[(30ft + 27ft)/2, 25ft]*12

$As_{,\ min}$ = 0.00075(2,736 in2) = 2.05 in^2 = 11 - #4 Top (2.20 in^2)

Exterior supports:

A_{cf} = max. (8in)[(27ft/2), 25ft]*12

$A_{s,\ min}$ = 0.00075(2,400 in^2) = 1.80 in^2

= 9 - #4 Top (1.80 in2)

Must span a minimum of 1/6 the clear span on each side of support (ACI 18.9.4.2)

At least 4 bars required in each direction (ACI 18.9.3.3)

Place top bars within 1.5h away from the face of the support on each side (ACI 18.9.3.3)

= 1.5 (8 in) = 12 in

Maximum bar spacing is 12” (ACI 18.9.3.3)

Check minimum reinforcement if it is sufficient for ultimate strength

$M_n = (A_s f_y + A_{ps} f_{ps})$ (d-a/2)

d = effective depth

$A_{ps} = 0.153in^2$*(number of tendons) = 0.153in^2*(20 tendons) = 3.06 in^2

$f_{ps} = f_{se} + 10{,}000 + (f'_c bd)/(300A_{ps})$ for slabs with L/h > 35 (ACI 18.7.2)

= 174,000psi + 10,000 + [(5,000psi)(25ft*12)d]/[(300)(3.06 in^2)]

= 184,000psi + 1634d

$a = (A_s f_y + A_{ps} f_{ps}) / (0.85f'_c b)$

At supports d = 8” - 3/4 ” - 1/4” = 7”

f_{ps} = 184,000psi + 1634(7”) = 195,438psi

a = [(2.20 in^2)(60 ksi) + (3.06 in^2)(195ksi)]/[(0.85)(5ksi)(25ft*12)] = 0.57

ϕMn = 0.9 [(2.20 in^2)(60 ksi) + (3.06 in^2)(195ksi)][7” - (0.57)/2]/12

= 0.9 (728k)(6.72in)/12 = 367 ft-k < 370ft-k

Reinforcement for ultimate strength requirements governs $As,_{reqd}$= 2.30in^2

12 - #4 Top at interior supports 9 - #4 Top at exterior supports

When reinforcement is provided to meet ultimate strength requirements, the minimum lengths must also conform to the provision of ACI 318-05 Chapter 12. (ACI 18.9.4.3)

At midspan (end span)

d = 8” - 11/2” - 1/4” = 6 1/4”

fps = 184,000psi + 1634(6.25”)

= 194,212psi

a= [(7.33 in2)(60 ksi) + (3.06 in^2)(194ksi)]/[(0.85)(5ksi)(25ft*12)] = 0.81

ϕMn = 0.9 [(7.33 in^2)(60 ksi) + (3.06 in^2)(194ksi)][6.25” - (0.81)/2]/12

= 0.9 (1033k)(5.85in)/12 = 453 ft-k > 289 ft-k

Minimum reinforcement ok

#5 @ 12“oc Bottom at end spans

This is a simplified hand calculation for a post-tensioned two-way plate design. A detailed example can be found in the PCA Notes on ACI 318-05 Building Code Requirements for Structural Concrete.

4. Circular prestressing

General

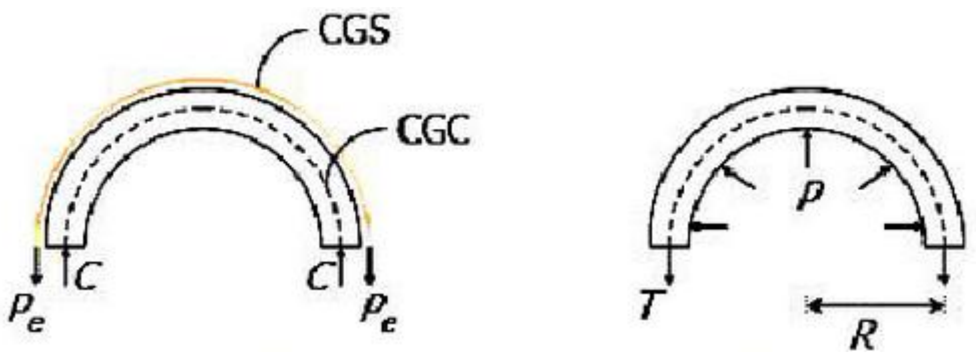
Circular Prestressing" is employed to denote the prestressing of circular structures such as pipes and tanks where the prestressing wires are wound in circles. In contrast to this term, "linear prestressing" is used to include all other types of prestressing, where the cables may be either straight or curved, but not wound in circles around a circular structure. In most prestressed circular structures, prestress is applied both circumferentially and longitudinally, the circumferential prestress being circular and the longitudinal prestress actually linear.

The circumferential prestressing resists the hoop tension generated due to the internal pressure. The prestressing is done by wires or tendons placed spirally, or over sectors of the circumference of the member. The wires or tendons lay outside the concrete core. Hence, the centre of the prestressing steel (CGS) is outside the core concrete section. When the prestressed members are curved, in the direction of prestressing, the prestressing is called circular prestressing. For example, circumferential prestressing in pipes, tanks, silos, containment structures and similar structures is a type of circular prestressing. In these structures, there can be prestressing in the longitudinal direction (parallel to axis) as well. Circular prestressing is also applied in domes and shells. [https://theconstructor.org]

Introduction

When the prestressed members are curved, in the direction of prestressing, the prestressing is called circular prestressing. For example, circumferential prestressing in pipes, tanks, silos, containment structures and similar structures is a type of circular prestressing. In these structures, there can be prestressing in the longitudinal direction (parallel to axis) as well. Circular prestressing is also applied in domes and shells. The circumferential prestressing resists the hoop tension generated due to the internal pressure. The prestressing is done by wires or tendons placed spirally, or over sectors of the circumference of the member. The wires or tendons lay outside the concrete core. Hence, the centre of the prestressing steel (CGS) is outside the core concrete section. The hoop compression generated is considered to be uniform across the thickness of a thin shell. Hence, the pressure line (or C-line) lies at the centre of the core concrete section (CGC). The following sketch shows the internal forces under service conditions. The analysis is done for a slice of unit length along the longitudinal direction (parallel to axis).

Liquid retaining structures, such as circular pipes, tanks and pressure vessels are admirably suited for circular prestressing. The circumferential hoop compression induced in concrete by prestressing counterbalances the hoop tension developed due to the internal fluid pressure. A reinforced concrete pressure pipe requires a large amount of reinforcement to ensure low-tensile stresses resulting in a crack-free structure. However, circular prestressing eliminates cracks and provides for an economical use of materials. In addition, prestressing safeguards against shrinkage cracks in liquid retaining structures. [https://www.scribd.com/document]



(a) Due to prestress (b) Due to internal pressure

Fig1: Internal forces under service conditions
[https://www.scribd.com/document]

To reduce the loss of prestress due to friction, the prestressing can be done over sectors of the circumference. Buttresses are usedfor the anchorage of the tendons. The following sketch shows the buttresses along the circumference.

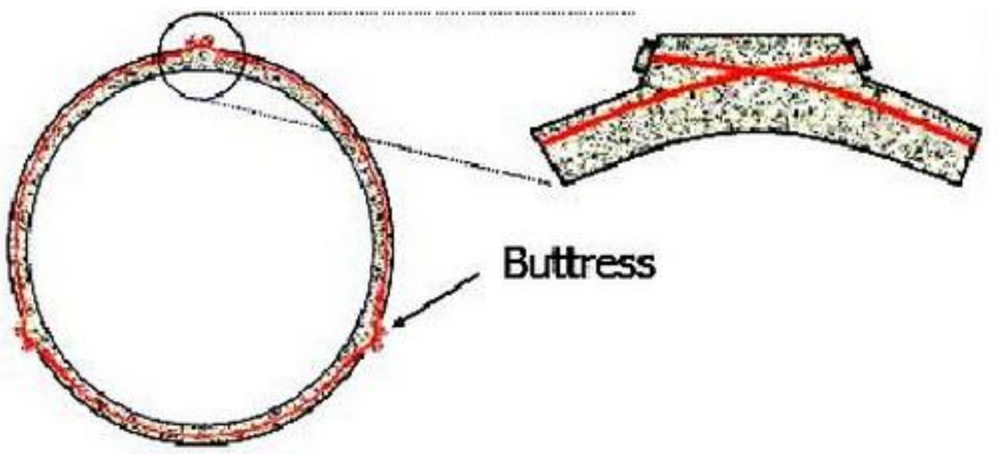


Fig 2: Use of buttress in Circular prestressing
[https://www.scribd.com/document]

Design parameters

General analysis

a) Analysis at transfer

The compressive stress can be calculated from the compression C. From equilibrium, C=Po, where P_o is the prestress at transfer after short-term losses. The compressive stress (f_c) is given as $f_c = -P_o /A$,
Where,
A=area of the longitudinal section of the slice. The permissible prestress is determined based on f_c within the allowable stress at transfer (f_{cc}/all).

b) Analysis at service loads

The tensile stress due to the internal pressure (p) can be calculated from the tension T .From equilibrium of half of the slice, T=ρR. Where, R is the radius of the mid-surface of the cylinder. The resultant stress (f_c) due to the effective prestress (P_e) and internal pressure is given as, $f_c = -P_o /A + \rho R/A_t$

A_t = area of the transformed longitudinal section of the slice.

Design

The internal pressure ρ and the radius are given variables. It is assumed that the prestressing steel alone carries the hoop tension due to internal pressure, that is,

$P_e = A_p f_{ps} = \rho R$.

The steps of design are as follows:

1) Calculate the area of the prestressing steel from the equation, $A_p= \rho R/f_{pe}$

2) Calculate the prestress at transfer from an estimate of the permissible initial stress f_{po} and using the equation $P_o= A_p f_{po}$.

3) Calculate the thickness of the concrete shell from the equation, A= Po/f_{cc}, here $f_{cc,all}$ is the allowable compressive stress at transfer.

4) Calculate the resultant stress f_c at the service condition. The value of f_c should be within $f_{cc,all}$ the allowable compressive stress at service conditions.

1) Design Example of a Post-tensioned Composite Bridge Girder

ACI-ASCE Committee 343, "Analysis and Design of Reinforced Concrete Bridge Structures." ACI Manual of Concrete Practice, Part 4, American Concrete Institute, Detroit, MI, 1989.

ASTM. (2006). "Standard specification for steel strand, uncoated seven-wire for pre-stressed concrete." A416/A416M-06, West Conshohocken, PA.

AASTHO (2007), "AASTHO LRFD Bridge Design Specifications." New York, Washington DC

2) DESIGN EXAMPLE OF A TWO-WAY POST-TENSIONED SLAB

Building code requirement for Structural Concrete, ACI 318-05 American Concrete Institute, 2005. [archive.org/stream/gov.law.aci.318.1995/aci.318.1995_djvu.txt]

Seismic Design of Precast concrete Building Structures, IBC-2003 (International Building Code 2003).

3) Circular prestressing

[https://theconstructor.org]; [https://www.scribd.com/document]